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ON LOCALLY COMPACT SHIFT-CONTINUOUS TOPOLOGIES ON SEMIGROUPS $\mathcal{C}_+(A, B)$ AND $\mathcal{C}_-(A, B)$ WITH ADJOINED ZERO

Let $\mathcal{C}_+(a, b)$ and $\mathcal{C}_-(a, b)$ be upper and down subsemigroups of the bicyclic semigroup defined in [15]. Let $\mathcal{C}_+(p, q)^0$ and $\mathcal{C}_-(p, q)^0$ be the semigroups $\mathcal{C}_+(a, b)$ and $\mathcal{C}_-(a, b)$ with the adjoined zero. We show that the semigroups $\mathcal{C}_+(p, q)^0$ and $\mathcal{C}_-(p, q)^0$ admit continuum many different Hausdorff locally compact shift-continuous topologies up to topological isomorphism.

Key words and phrases: semitopological semigroup, topological semigroup, left topological semigroup, locally compact.

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In this paper we shall follow the terminology of [4, 5, 6, 18].

By ω we denote the set of all non-negative integers. Throughout these notes we always assume that all topological spaces involved are Hausdorff.

Definition 1 ([4, 18]). *Let S be a non-void topological space which is provided with an associative multiplication (a semigroup operation) $\mu: S \times S \rightarrow S$, $(x, y) \mapsto \mu(x, y) = xy$. Then the pair (S, μ) is called*

- (i) *a right topological (left topological) semigroup if all interior left (right) shifts $\lambda_s: S \rightarrow S$, $x \mapsto sx$ ($\rho_s: S \rightarrow S$, $x \mapsto xs$), are continuous maps, $s \in S$;*
- (ii) *a semitopological semigroup if the map μ is separately continuous;*
- (iii) *a topological semigroup if the map μ is jointly continuous.*

We usually omit the reference to μ and write simply S instead of (S, μ) . It goes without saying that every topological semigroup is also semitopological and every semitopological semigroup is both a right and left topological semigroup.

A topology τ on a semigroup S is called:

- a *semigroup topology* if (S, τ) is a topological semigroup;

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- a *shift-continuous* topology if (S, τ) is a semitopological semigroup;
- an *left-continuous* (*right-continuous*) topology if (S, τ) is a left (right) topological semigroup.

The bicyclic monoid $\mathcal{C}(a, b)$ is the semigroup with the identity 1 generated by two elements a and b subjected only to the condition $ab = 1$. The semigroup operation on $\mathcal{C}(a, b)$ is determined as follows:

$$b^k a^l \cdot b^m a^n = \begin{cases} b^{k-l+m} a^n, & \text{if } l < m; \\ b^k a^n, & \text{if } l = m; \\ b^k a^{l-m+n}, & \text{if } l > m. \end{cases}$$

In [15] Makanjuola and Umar study algebraic property of the following anti-isomorphic subsemigroups

$$\mathcal{C}_+(p, q) = \{q^i p^j \in \mathcal{C}(p, q) : i \leq j\} \quad \text{and} \quad \mathcal{C}_-(p, q) = \{q^i p^j \in \mathcal{C}(p, q) : i \geq j\},$$

of the bicyclic monoid. In the paper [8] we prove that every Hausdorff left-continuous (right-continuous) topology on the monoid $\mathcal{C}_+(a, b)$ ($\mathcal{C}_-(a, b)$) is discrete and show that there exists a compact Hausdorff topological monoid S which contains $\mathcal{C}_+(a, b)$ ($\mathcal{C}_-(a, b)$) as a submonoid. Also, in [8] we constructed a non-discrete right-continuous (left-continuous) topology τ_p^+ (τ_p^-) on the semigroup $\mathcal{C}_+(a, b)$ ($\mathcal{C}_-(a, b)$) which is not left-continuous (right-continuous).

Later by $\mathcal{C}_+(p, q)^0$ and $\mathcal{C}_-(p, q)^0$ we denote the semigroups $\mathcal{C}_+(a, b)$ and $\mathcal{C}_-(a, b)$ with the adjoined zero.

In [7] it is proved that every Hausdorff locally compact shift-continuous topology on the bicyclic monoid with adjoined zero is either compact or discrete. This result was extended by Bardyla onto the p -polycyclic monoid [1] and graph inverse semigroups [2], and by Mokrytskyi onto the monoid of order isomorphisms between principal filters of $\mathbb{N}^n$ with adjoined zero [17]. In [9] the results of paper [7] onto the monoid $\mathbf{IN}_\infty$ of all partial cofinite isometries of positive integers with adjoined zero are extended. In [12] the similar dichotomy was proved for so called bicyclic extensions $\mathbf{B}_\omega^\mathcal{F}$ when a family $\mathcal{F}$ consists of inductive non-empty subsets of ω . Algebraic properties on a group G such that if the discrete group G has these properties, then every locally compact shift continuous topology on G with adjoined zero is either compact or discrete studied in [16]. The above results are extended in [10] to the bicyclic extension $\mathbf{B}_{[0, \infty)}$ of the additive group of reals with adjoined zero (see [13]) in the cases when on the semigroup $\mathbf{B}_{[0, \infty)}$ the usual topology, the discrete topology or the topology determined by the natural partial order is defined. Also, in [11] it is proved that the extended bicyclic semigroup $\mathcal{C}_\mathbb{Z}^0$ with adjoined zero admits distinct $\mathfrak{c}$ -many shift-continuous topologies, however every Hausdorff locally compact semigroup topology on $\mathcal{C}_\mathbb{Z}^0$ is discrete. In [3] Bardyla proved that a Hausdorff locally compact semitopological semigroup McAlister Semigroup $\mathcal{M}_1$ is either compact or discrete. However, this dichotomy does not hold for the McAlister Semigroup $\mathcal{M}_2$ and moreover, $\mathcal{M}_2$ admits continuum many different Hausdorff locally compact inverse semigroup topologies [3].

In this paper we show that the semigroups $\mathcal{C}_+(p, q)^0$ and $\mathcal{C}_-(p, q)^0$ admit continuum many different Hausdorff locally compact shift-continuous topologies up to topological isomorphism.

Lemma 1. *Every locally compact Hausdorff shift-continuous topology τ on the additive semigroup of non-negative integers $(\omega, +)$ is discrete.*

Proof. Fix any $n_0 \in \omega$. The Hausdorffness of the space (ω, τ) implies that $n_0^\downarrow = \{k \in \omega : k \leq n_0\}$ is a closed subset of (ω, τ) . Then $\omega \setminus n_0^\downarrow$ is an open subset of (ω, τ) , and by Corollary 3.3.10 of [6], $\omega \setminus n_0^\downarrow$ is locally compact, and hence, Baire. By Proposition 1.30 of [14] the space $\omega \setminus n_0^\downarrow$ contains an isolated point n_1 , which is isolated in (ω, τ) because $\omega \setminus n_0^\downarrow$ is an open subset of (ω, τ) . This and the condition $n_0 < n_1$ imply that n_0 is an isolated point in (ω, τ) , because n_0 is the full preimage of n_1 under the continuous right shift $\rho_{n_1-n_0} : (\omega, +, \tau) \rightarrow (\omega, +, \tau)$, $i \mapsto i + (n_1 - n_0)$. This completes the proof of the lemma. $\square$

Later by $(\omega, +)^0$ we denote the additive semigroup of non-negative integers $(\omega, +)$ with adjoined zero. Without loss of generality we may assume that $(\omega, +)^0 = \omega \cup \{\infty\}$ with the extended semigroup operation $n + \infty = \infty + n = \infty + \infty = \infty$ for all $n \in \omega$, i.e., ∞ is the zero of $(\omega, +)^0$.

Proposition 1. *Every Hausdorff locally compact shift-continuous topology on the semigroup $(\omega, +)^0$ is either compact or discrete.*

Proof. Let τ_{lc} be an arbitrary non-discrete Hausdorff locally compact shift-continuous topology on the semigroup $(\omega, +)^0$. The Hausdorffness of $((\omega, +)^0, \tau_{lc})$ implies that ω is an open subset of $((\omega, +)^0, \tau_{lc})$. Then by Corollary 3.3.10 of [6], ω is locally compact, and by Lemma 1 is a discrete subspace of $((\omega, +)^0, \tau_{lc})$.

Since all point from ω are open-and-closed subsets of the locally compact space $((\omega, +)^0, \tau_{lc})$, there exists a base $\mathcal{B}_{\tau_{lc}}(\infty)$ of the topology τ_{lc} at the point ∞ which consists of compact-and-open subsets of $((\omega, +)^0, \tau_{lc})$. Hence, for any $U, V \in \mathcal{B}_{\tau_{lc}}(\infty)$ the set $U \setminus V$ is finite.

We state that for any $U \in \mathcal{B}_{\tau_{lc}}(\infty)$ the set $\omega \setminus U$ is finite. Suppose to the contrary that there exists $U \in \mathcal{B}_{\tau_{lc}}(\infty)$ the set $\omega \setminus U$ is infinite. The separate continuity of the semigroup operation in $((\omega, +)^0, \tau_{lc})$ implies that there exists $V \in \mathcal{B}_{\tau_{lc}}(\infty)$ such that $V \subseteq U$ and $1 + V \subseteq U$. Since $\omega \setminus U$ is infinite, there exists a sequence $\{x_n\}_{n \in \omega} \subseteq U$ such that $1 + x_i \notin U$ for any $i \in \omega$. This implies that $x_i \notin V$ for any $i \in \omega$, and hence, the set $U \setminus V$ is infinite, a contradiction. The obtained contradiction implies that τ_{lc} is a compact topology on $(\omega, +)^0$. $\square$

Later by τ_{lc} we denote a Hausdorff locally compact shift-continuous topology on the semigroup $\mathcal{C}_+(p, q)^0$.

Since every Hausdorff shift-continuous topology on the semigroup $\mathcal{C}_+(p, q)$ is discrete (see [8, Theorem 6]), the following statements holds.

Lemma 2. *If U and V are any compact-and-open neighbourhoods of the zero in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$, then the set $U \setminus V$ is finite.*

For any $i \in \omega$ we denote

$$\mathcal{C}_+^i(p, q) = \{b^i a^{i+s} \in \mathcal{C}_+(p, q) : s \in \omega\}.$$

The semigroup operation of $\mathcal{C}_+(p, q)$ implies that $\mathcal{C}_+^i(p, q)$ is a subsemigroup of $\mathcal{C}_+(p, q)$, and moreover, $\mathcal{C}_+^i(p, q)$ is isomorphic to the additive semigroup of non-negative integers $(\omega, +)$ for any $i \in \omega$ [8].

Lemma 3. *For any compact-and-open neighbourhood U of the zero in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ there exists $i \in \omega$ such that the set $U \cap \mathcal{C}_+^i(p, q)$ is infinite.*

Proof. Suppose to the contrary that there exists a compact-and-open neighbourhood U of the zero in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ such that $|U \cap \mathcal{C}_+^i(p, q)| < \infty$ for any $i \in \omega$. Then there exists a sequence $\{i_j\}_{j \in \omega} \subseteq \omega$ such that $U \cap \mathcal{C}_+^{i_j}(p, q) \neq \emptyset$ for any $j \in \omega$. The separate continuity of the semigroup operation in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ and local compactness of τ_{lc} imply that there exists a compact-and-open neighbourhood V of zero in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ such that $V \subseteq U$ and $V \cdot a \subseteq U$. By the definition of the semigroup operation in $\mathcal{C}_+(p, q)$ we get that $\mathcal{C}_+^i(p, q) \cdot a \subseteq \mathcal{C}_+^i(p, q)$ for all $i \in \omega$. Since for any $j \in \omega$ the set $U \cap \mathcal{C}_+^{i_j}(p, q)$ is non-empty and finite, there exists maximal non-negative integer s_j such that $b^{i_j} a^{i_j+s_j} \in U$ but $b^{i_j} a^{i_j+s_j+1} \notin U$. This implies that the set $U \setminus V$ is infinite, which contradicts Lemma 2. The obtained contradiction implies the statement of the lemma. $\square$

Lemma 4. *For any compact-and-open neighbourhood U of the zero in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ there exists $i_0 \in \omega$ such that $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ is a compact subset of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$.*

Proof. By Lemma 3 for any compact-and-open neighbourhood U of the zero in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ there exists $i_0 \in \omega$ such that the set $U \cap \mathcal{C}_+^{i_0}(p, q)$ is infinite. Since $\mathcal{C}_+(p, q)$ is a discrete subspace of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$, $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ is a closed subset of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$. By Corollary 3.3.10 of [6], $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ is locally compact. Since the semigroup $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ is isomorphic to the additive semigroup of non-negative integers with adjoined zero $(\omega, +)^0$, by Proposition 1 the semigroup $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ is a compact subsemigroup of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$. $\square$

Lemma 5. *$\mathcal{C}_+^i(p, q) \cup \{0\}$ is a compact subset of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ for any $i \in \omega$.*

Proof. By Lemma 4 there exists $i_0 \in \omega$ such that $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ is a compact subset of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$. We fix an arbitrary $i \in \omega$. The semigroup operation in $\mathcal{C}_+(p, q)^0$ implies the following:

(1) if $i < i_0$, then

$$\begin{aligned} a^{i_0-i} \cdot \mathcal{C}_+^{i_0}(p, q) &= \{a^{i_0-i} \cdot b^{i_0} a^{i_0+s} : s \in \omega\} = \\ &= \{b^{i_0-(i_0-i)} a^{i_0+s} : s \in \omega\} = \\ &= \{b^i a^{i_0+s} : s \in \omega\} = \\ &= \mathcal{C}_+^i(p, q) \setminus \{b^i a^i, \dots, b^i a^{i_0-1}\}; \end{aligned}$$

(2) if $i > i_0$, then

$$\begin{aligned} b^i a^i \cdot \mathcal{C}_+^{i_0}(p, q) &= \{b^i a^i \cdot b^{i_0} a^{i_0+s} : s \in \omega\} = \\ &= \{b^i a^i \cdot b^{i_0} a^{i_0} \cdot a^s : s \in \omega\} = \\ &= \{b^i a^i \cdot a^s : s \in \omega\} = \\ &= \{b^i a^{i+s} : s \in \omega\} = \\ &= \mathcal{C}_+^i(p, q). \end{aligned}$$

Hence, if $i > i_0$, then $\mathcal{C}_+^i(p, q) \cup \{0\}$ is a compact subset of $(\mathcal{C}_+(p, q)^0, \tau_{lc})$ as a continuous image of compact space $\mathcal{C}_+^{i_0}(p, q) \cup \{0\}$ under the left shift $\lambda_{b^i a^i} : x \mapsto b^i a^i \cdot x$ in $(\mathcal{C}_+(p, q)^0, \tau_{lc})$. By a similar way, in the case when $i < i_0$ we obtain that $\mathcal{C}_+^i(p, q) \cup \{0\}$ is compact, because it is the union of the finite family of compact subsets $\{a^{i_0-i} \cdot \mathcal{C}_+^{i_0}(p, q), \{b^i a^i\}, \dots, \{b^i a^{i_0-1}\}\}$. $\square$

Example 1. Let $\{x_i\}_{i \in \omega}$ be any non-decreasing sequence of non-negative integers. We define the topology $\tau_{\{x_i\}}$ on the semigroup $\mathcal{C}_+(p, q)^0$ in the following way. Put

$$U_{\{x_i\}}^n(0) = \{0\} \cup \{b^k a^{k+x_k+s} : k, s \in \omega \text{ and } k + x_k + s > n\}.$$

We suppose that all points of the set $\mathcal{C}_+(p, q)$ are isolated in $(\mathcal{C}_+(p, q)^0, \tau_{\{x_i\}})$, and the family $\mathcal{B}_{\{x_i\}}(0) = \{U_{\{x_i\}}^n(0) : n \in \omega\}$ is the base of the topology $\tau_{\{x_i\}}$ at zero 0 of the semigroup $\mathcal{C}_+(p, q)^0$.

It is obvious that the space $(\mathcal{C}_+(p, q)^0, \tau_{\{x_i\}})$ is Hausdorff and locally compact.

Next we show that the semigroup operation in $(\mathcal{C}_+(p, q)^0, \tau_{\{x_i\}})$ is separately continuous.

Suppose $b^m a^{m+x_m+s} \in U_{\{x_i\}}^n(0)$ and $m \leq n = i + j \geq i$. Then $m + x_m + s > n$, and hence, $i + j + x_m + s \geq n$, which implies that $b^i a^{i+j+x_m+s} \in U_{\{x_i\}}^n(0)$.

If $m > n = i + j \geq i$, then $b^i a^{i+j} \cdot b^m a^{m+x_m+s} = b^{m-j} a^{m+x_m+s}$. In the case when $m - j \leq n$ we have that $m + x_m + s \geq n$ and $b^{m-j} a^{m+x_m+s} \in U_{\{x_i\}}^n(0)$. In the case when $m - j > n$ we have that

$$m - j + x_{m-j} + s \leq m - j + x_m + s \leq m + x_m + s,$$

because $\{x_i\}_{i \in \omega}$ is a non-decreasing sequence of non-negative integers, and hence $b^{m-j} a^{m+x_m+s} \in U_{\{x_i\}}^n(0)$. Therefore, the inclusion $b^i a^{i+j} \cdot U_{\{x_i\}}^n(0) \subseteq U_{\{x_i\}}^n(0)$ holds for any $n \geq i + j$.

If $m \geq n$, then $m + x_m + s > n$, and hence, we have that

$$b^m a^{m+x_m+s} \cdot b^i a^{i+j} = b^m a^{m+x_m+s-i+i+j} = b^m a^{m+j+x_m+s}.$$

This implies that for any $n \geq i + j$ the following inclusion $U_{\{x_i\}}^n(0) \cdot b^i a^{i+j} \subseteq U_{\{x_i\}}^n(0)$ holds.

Therefore, the semigroup operation in $(\mathcal{C}_+(p, q)^0, \tau_{\{x_i\}})$ is separately continuous.

Since there exist continuum many non-decreasing sequence of non-negative integers in ω , Lemma 5 and Example 1 imply the main theorem of this paper.

Theorem 1. On the semigroup $\mathcal{C}_+(p, q)^0$ ($\mathcal{C}_-(p, q)^0$) there exist continuum many Hausdorff locally compact shift-continuous topologies up to topological isomorphism.

Since for any non-decreasing sequence of non-negative integers $\{x_i\}_{i \in \omega}$ in ω and any $n \in \omega$ the set $\mathcal{C}_+(p, q)^0 \setminus U_{\{x_i\}}^n(0)$ is either finite or infinite, we get the following corollary.

Corollary 1. On the semigroup $\mathcal{C}_+(p, q)^0$ ($\mathcal{C}_-(p, q)^0$) there exist exactly three Hausdorff locally compact shift-continuous topologies up to homeomorphism.

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Гутік О.В. *Про локально компактні трансляційно неперервні топології на напівгрупах $\mathcal{C}_+(a, b)$ і $\mathcal{C}_-(a, b)$ з приєднаним нулем* // Буковинський матем. журнал — 2024. — Т.12, №1. — С. 14–20.

У праці [15] Маканюола та Умар вивчали алгебричні властивості верхньої $\mathcal{C}_+(a, b) = \{q^i p^j \in \mathcal{C}(p, q) : i \leq j\}$ та нижньої $\mathcal{C}_-(a, b) = \{q^i p^j \in \mathcal{C}(p, q) : i \geq j\}$ піднапівгруп біциклічного моноїда $\mathcal{C}(a, b)$. Прийmemo $\mathcal{C}_+(p, q)^0$ і $\mathcal{C}_-(p, q)^0$ – напівгрупи $\mathcal{C}_+(a, b)$ і $\mathcal{C}_-(a, b)$ з приєднаним нулем. Відомо [7], що на біциклічній напівгрупі з приєднаним нулем $\mathcal{C}(p, q)^0$ кожна гаусдорфова локально компактна трансляційно неперервна топологія є або компактною, або дискретною. У цій праці описано всі гаусдорфові локально компактні трансляційно неперервні топології на адитивній напівгрупі невід’ємних цілих чисел з приєднаним нулем $(\omega, +)^0$ і на напівгрупах $\mathcal{C}_+(p, q)^0$ і $\mathcal{C}_-(p, q)^0$. Зокрема доведено, що на напівгрупах $\mathcal{C}_+(p, q)^0$ і $\mathcal{C}_-(p, q)^0$ існує континуум різних гаусдорфових локально компактних трансляційно неперервних топологій з точністю до топологічного ізоморфізму, причому таких топологій на напівгрупах $\mathcal{C}_+(p, q)^0$ і $\mathcal{C}_-(p, q)^0$ існує рівно три з точністю до гомеоморфізму.